

**HOMEWORK 5**

**Que1:** (20 p) Calculate the integrals given below. (Hint: Consider Euler expansion). Assume that  $a \geq 0$  and  $b \geq 0$

a-)  $\int_0^\infty \frac{x^a}{(x^2 + 1)^2} dx : \quad (-1 < a < 3), \text{ where } x^a = \exp(a \ln x).$

b-)  $\int_{-\infty}^\infty \frac{x^3 \sin ax}{x^4 + 4} dx$

c-)  $\int_0^{2\pi} \frac{dx}{2+\cos(x)}$

d-)  $\int_0^{2\pi} \frac{dx}{(2+2\sin(x))^2}$

**Que2:** (20p) Calculate the integrals given below by using contour. Consider,  $\int_0^\infty \frac{\log(z)}{z^2+4} dz$

a-)  $\int_0^\infty \frac{\log(x)}{x^2+4} dx$

b-)  $\int_0^\infty \frac{\log^2(x)}{x^2+4} dx$

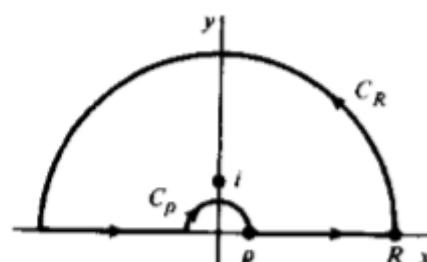


Figure 1: Contour

**Que3:** (30 p)

Find the series representation of

a-)  $f(z) = -\frac{1}{(z-1)(z-2)}$  for  $|z|<1$ ,  $1<|z|<2$  and  $|z|>2$  regions

b-)  $f(z) = \frac{1}{z^2+3z+2}$  for  $|z|<1$ ,  $1<|z|<2$  and  $|z|>2$  regions

c-)

Write the two Laurent series in powers of  $z$  that represent the function

$$f(z) = \frac{1}{z(1 + z^2)}$$

in certain domains, and specify those domains.

d-)

Give two Laurent series expansions in powers of  $z$  for the function

$$f(z) = \frac{1}{z^2(1 - z)},$$

and specify the regions in which those expansions are valid.

e-) Derive the laurent series of :

$$z^3 \cosh\left(\frac{1}{z}\right)$$

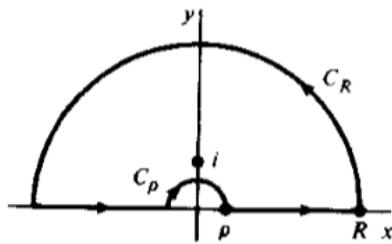
**Que4:(30p)**

a-)

- (a) By integrating the function  $f(z) = e^{iz}/z$  around the simple closed contour in Fig. 58, show that

$$2i \int_{\rho}^R \frac{\sin x}{x} dx = - \int_{C_p} f(z) dz - \int_{C_R} f(z) dz,$$

where  $C_p$  and  $C_R$  are the arcs indicated in that figure.

**Figure 58**

b-)



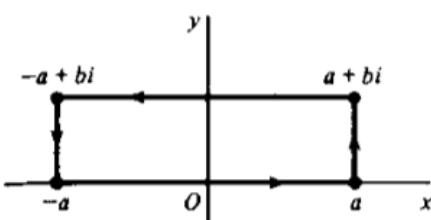
17. Given that (see the footnote to Exercise 11)

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2},$$

derive the integration formula

$$\int_0^\infty \exp(-x^2) \cos(2bx) dx = \frac{\sqrt{\pi}}{2} \exp(-b^2) \quad (b > 0)$$

by integrating the function  $\exp(-z^2)$  around the rectangular path shown in Fig. 59 and then letting  $a$  tend to infinity.

**Figure 59**

c-)

- (a) By considering the integral of  $\exp(iz^2)$  around the positively oriented boundary of the sector  $0 \leq r \leq R, 0 \leq \theta \leq \pi/4$ , show that

$$\int_0^R e^{iz^2} dx = e^{i\pi/4} \int_0^R e^{-r^2} dr - \int_{C_R} e^{iz^2} dz,$$

where  $C_R$  is the arc  $z = Re^{i\theta}$  ( $0 \leq \theta \leq \pi/4$ ).

